

# **RECORD PROCESSING CONSIDERATIONS FOR THE EFFECTS OF TILTING AND TRANSIENTS**

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## **ABSTRACT**

In the near-field of an earthquake rotational component of ground motion may not be negligible as compared to acceleration of translational motion. Analysis of the equation of motion of horizontal and vertical pendulums has shown that horizontal sensors are sensitive not only to translational motion but also to tilts. Ignoring tilt sensitivity may produce unreliable results, especially in residual displacement and long-period calculations. Numerical tests were performed to evaluate effect of instrument tilting on displacement calculations. In contrast to horizontal sensors, vertical sensors don't have these limitations, since they are much less sensitive to tilts. In general, only six-component systems measuring rotations and accelerations, or three-component systems similar to systems used in inertial navigation assuring purely translational motion of accelerometers can be used to calculate residual displacements.

## **INTRODUCTION**

Strong-motion data are used in seismological studies and in earthquake engineering. For many years from its birth in the 1930s, strong-motion seismology was mostly oriented toward earthquake engineering with very little impact on seismology. On the other hand, strong-motion also means that records are obtained in the near-field of an earthquake or an explosion. Seismologists started using near-field strong-motion records as a tool of studying the earthquake source process. Obviously, compared to teleseismic records, near-field data present an opportunity to take a close look at a seismic source with much less distortion by the wave propagation path. Source studies pushed investigators to look at the possibilities of getting more information from the records than allowed by classical strong-motion data processing [1-3]. New methods of data processing were developed, allowing determination of ground motion including residual displacement from accelerograms [4-7].

Strong-motion seismology employed the same pendulum type instruments as used in classical seismology. The main differences between strong-motion and weak-motion seismometers are the following:

1. Strong-motion instruments are much less sensitive to the ground motion.
2. Their output is proportional to the acceleration as oppose to velocity or displacement sensors used in classical seismology.

All assumptions used in classical weak-motion seismology were simply transposed onto the area of strong-motion. The most questionable perception brought from classical seismology into strong-motion is the assumption of simple linear input motion of the ground with rotational (tilt) component being negligible relative to the linear one.

Ignoring rotation has two consequences:

1. We still have very primitive knowledge about this important component of strong ground motion, with mostly theoretical or indirect assessments of rotational component [8-12].
2. We approximate the output of the instrument as acceleration. Subsequently, by integrating this signal we assume getting velocity and displacement.

In real near-field of an earthquake, input of the rotational component may not be negligible as compared to acceleration of the linear motion. As a result, records that are assumed to be accelerations may actually be a sum of acceleration and tilt (It is well-known that the same type of pendulums can also be used as tiltmeters in the low frequency range).

Evidently, in the near-field of an earthquake it is more appropriate to measure all six components of the motion: 3 linear and 3 rotational. There are different possible ways of dealing with this situation [13-20]. The general solution is to combine 3 linear motion sensors with 3 rotational ones at the same point of measurement. Another way of dealing with the problem is to assure that the seismic sensors are moving strictly linearly in space, for example, by using a gyroscopic platform (similar to the inertial navigation).

These issues become especially important now because of new trends in technology and data processing. Recently developed high-resolution digital accelerographs provide new possibilities in data processing, but they should be applied carefully with full understanding of the basics and possible errors of data recording. Different group of researchers apply various techniques of acceleration data processing including residual displacement calculations. In our opinion, it can be done only if certain conditions apply.

## THEORY OF THE PENDULUM

To avoid wrong assumptions, let's start with examining the differential equation of pendulum motion. This step looks obvious, but actually deserve special attention. Looking at the basic equation of pendulum motion we discovered the following interesting fact: It is written differently in different classical seismological sources (Golitsyn [13], Rodgers [21], Aki & Richards [22]).

The differential equation of a horizontal pendulum oscillating in a horizontal plane can be written as:

$$L: y_1'' + 2\omega_1 D_1 y_1' + \omega_1^2 y_1 = -x_1'' + g\psi_2 - \psi_3'' l_1 + x_2'' \theta_1 \quad (1a)$$

$$T: y_2'' + 2\omega_2 D_2 y_2' + \omega_2^2 y_2 = -x_2'' + g\psi_1 + \psi_3'' l_2 + x_1'' \theta_2 \quad (1b)$$

Where:

- $y_i$  is recorded response of the instrument,
- $\theta_i$  is the angle of pendulum rotation,

$l_i$  is the length of pendulum arm,

$$y_i = \theta_i l_i,$$

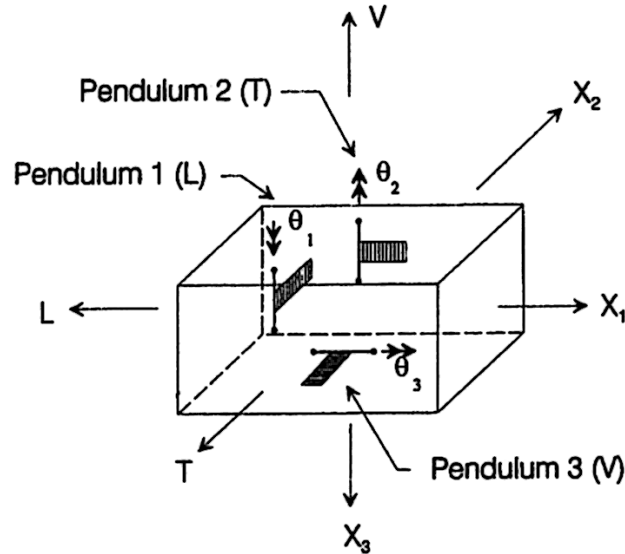
$\omega_i$  and  $D_i$  are respectively the natural frequency and fraction of critical damping of the  $i$ th transducer,

$g$  is acceleration due to gravity,

$x_i''$  is ground acceleration in  $i$ th direction,

$\psi_i$  is a rotation of the ground surface about  $x_i$  axis.

Equations (1a,b) for the two horizontal directions L and T describe pendulum response to low amplitude motion when  $\sin(\psi_i) \approx \psi_i$ . Figure 1 shows schematic representation of an accelerograph.



**Figure 1. Schematic representation of three transducers in an accelerograph. The coordinate axes  $X_1$ ,  $X_2$  and  $X_3$  serve to describe the motion of the L, T and V transducers respectively. Angles  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  describe the deflection of the transducer pendulums (Modified from Trifunac and Todorovska [23]).**

Sensitivity of vertical pendulum to tilts is different. For small tilts it is proportional to  $(1 - \cos(\psi))$  and  $\cos(\psi) \approx 1 - \psi^2/2$ .

The equation of the vertical pendulum can be written as follows:

$$V: y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' + g\psi_1^2/2 - \psi_1'' l_3 + x_2'' \theta_3$$

Vertical pendulum is sensitive to the vertical acceleration, angular acceleration, cross axis motion, but is much less sensitive to tilts (for small tilts).

Neglecting  $g\psi_1^2/2$  will give us:

$$V: y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' - \psi_1'' l_3 - x_2'' \theta_3 \quad (1c)$$

This means that a horizontal pendulum (1a or 1b) is sensitive to the acceleration of linear motion, tilt, and angular acceleration and has also cross axis sensitivity. Regrettably, the completeness of representing eq.1 in the literature varies. For example, Golitsyn (1912) [13] does not take into account the cross-axis sensitivity, while Aki and Richards (1980) [22] ignore the angular acceleration term.

It is critical to study the sensitivity of a pendulum to the second, third and fourth items in equations (1a,b) and (1c). In teleseismic studies using typical seismometers, the effect of these items was considered to be small enough to be neglected. The questions are: Is it also true for the strong-motion near-field studies? If the answer is “No”, which items in the right part of the equation influence the output of the strong-motion instrument?

Possible impacts of different terms in the right part of equation (1) were studied by Trifunac, Wong and Todorovska [21-23] and Graizer [18]. Based on numerical simulations performed for a number of typical strong-motion instruments, Graizer [18] concluded that tilts could significantly impact output of the horizontal instrument. The effect of angular acceleration is significant for instruments with a relatively long pendulum arm, like a classical seismometer, but is relatively small for typical accelerometers with a short pendulum arm. The effect of cross-axis sensitivity may reach a few percent for motions higher than 2 g for accelerometers with a natural frequency of 25 Hz. It is almost negligible for modern accelerometers with a natural frequency of about 100 Hz.

As it was summarized by Trifunac and Todorovska [23], for some instruments the cross-axis sensitivity term may be negligible, while for others  $l_i = 0$  (pendulum of spring-mass type). The terms caused by tilting are always present for the horizontal pendulum, and cannot be neglected.

For small oscillations, the vertical seismometer is almost not sensitive to tilts (of course this fact was well known to instrument designers long before). Taking into account these results, differential equations of the horizontal and vertical pendulums can be approximated:

$$L: \quad y_1'' + 2\omega_1 D_1 y_1' + \omega_1^2 y_1 = -x_1'' + g\psi_2 \quad (2a)$$

$$T: \quad y_2'' + 2\omega_2 D_2 y_2' + \omega_2^2 y_2 = -x_2'' + g\psi_1 \quad (2b)$$

$$V: \quad y_3'' + 2\omega_3 D_3 y_3' + \omega_3^2 y_3 = -x_3'' \quad (2c)$$

This means that in a typical strong-motion triaxial system, the two horizontal sensors are responding to the combination of horizontal accelerations and tilts, and the vertical sensor is only responding to the vertical acceleration.

This result may have very important consequences and also raise questions when dealing with strong-motion records from earthquakes.

- What is responsible for the differences between horizontal and vertical components in the long-period area during a real earthquake?
- Is it a principal difference in spectral contents of horizontal and vertical components, or does this difference actually represent error in recording acceleration contaminated with tilt at horizontal components?

The horizontal sensor, as one can see from the equation (2a, b) is sensitive to the second derivative of displacement and to tilt. This means that double integration of the equation (2a or b) will produce the sum of displacement and double integrated tilt.

Assuming that tilt is proportional to velocity (Trifunac, Todorovska [23]), double integration will give results proportional to integral of displacement. It can look like long-period noise on a result. Processing (integrating) a record of a horizontal sensor may result in long-period contamination of horizontal displacement.

Based on equation (2) we suggest performing a simple testing of the 3-component systems for the strongest records: Compare long-period components of the two horizontal and vertical records. If their levels are of the same order, this can give us more confidence in attributing long periods to ground displacement. If the level of long-period motion is significantly higher in the horizontal components, it can possibly be due to tilts.

### **RESIDUAL DISPLACEMENTS AND WHAT CAN BE DONE IN ABSENCE OF ROTATIONS (TILTS)**

Let's look at the differential equation of pendulum motion in absence of rotations. In this case equations (2a, b) can be simplified and will be similar to the equation (2c) for both vertical and horizontal components:

$$y'' + 2\omega D y' + \omega^2 y = -Vx'', \quad (3)$$

where  $V$  represents a magnification factor. Correspondently, ground displacement  $x(t)$  can be found by integrating recorded output of the instrument  $y(t)$ .

The first algorithm to compute residual ground displacement from recorded strong motion accelerograms appears to have been by Bogdanov and Graizer [4] later modified by Graizer [5]. A key part of the proposed method is baseline correction that can be accomplished by minimizing the functional ( $W$ ) based on realistic assumptions of minimum velocity at the beginning and at the end of an earthquake ground motion:

$$W = \int_0^{T_1} [x'(t)]^2 dt + \int_{T_2}^T [x'(t)]^2 dt \quad (4)$$

where  $T$  is the length of the recorded signal and  $T_1$  and  $T_2$  are times such that  $0 < T_1 < T_2 < T$ . This approach is based on the assumption that time intervals  $[0, T_1]$  and  $[T_2, T]$  can be found during which the ground motion is small compared to the strong motion

amplitudes [5]. A baseline was first approximated by polynomials of up to the third, and later up to the higher degrees. In real applications of this method polynomials of 2<sup>nd</sup> to 5<sup>th</sup> degrees were used for baseline correction.

Of course, the most challenging part was to convince the seismological community in our ability to recover residual displacement from records of real instruments. The first series of tests were performed with Soviet film accelerograph (very similar to the SMA-1). Similar tests were performed in 1991 at the Institut de Physique du Globe (Strasbourg, France) and in 1993 with the Kinematics FBA, Terra Tech SSA-302, Sundstrand SSD3 and Teledyne SA-220 sensors with W. Lee in the U.S. Geological Survey in Menlo Park (1993). Those instruments were put on a shake-table (or a specially designed cart), and their residual displacements were independently recorded by a ruler or a special gauge. Results of these tests were published in a series of papers [4,5] and reports. This is the most convincing classical way of testing methods of data processing. Shake-table tests clearly proved that residual displacement could be recovered from the record.

During the first series of experiments performed at the end of the 1970s we “discovered” that if the instrument is even slightly tilted during its movement, it makes recovery of residual displacement almost impossible. First, a series of tests were simply performed by sliding the accelerograph along the surface of the table. In this case, to overcome the static friction at the beginning of the motion, tilting of the instrument may occur. To avoid this problem a cart or shake-table was used in later tests (Graizer [5,18,19]).

Comparisons were also made of results obtained from different instruments installed at the same place.

Another group of tests was performed by using numerical testing of the algorithm. In this case the ideal (calculated) response of the instrument was distorted by systematic and random errors. This group of tests is extremely important because it allows studying the effect of each factor separately. It also allowed formulating the requirements for the quality (dynamic range) of the records necessary to obtain residuals.

Numerical tests also led to another important result: Random errors in acceleration can result in long-period disturbance after double integration [5].

Summarizing the results of all these tests, one can conclude that processing real accelerograms to get true ground displacement including residual requires satisfaction of a set of conditions:

1. The input ground motion is purely translational, without any tilting or any other natural distortions.
2. The record contains clear beginning and ending parts with relatively low level amplitudes to allow for baseline correction.
3. The signal to noise level of the record should be high enough, at least 40 Db.

Compliance with the second and third conditions is usually possible (and can be checked), especially for digital records. But the first condition cannot be checked unless independent measurements of rotations are performed.

## NUMERICAL TESTING OF TILT EFFECT ON DISPLACEMENT CALCULATION

A number of numerical tests were performed to study the influence of tilt on the ability of displacement calculations including a residual.

Figure 2 shows the first test in which the acceleration record was contaminated by tilt. The 1st test record was produced by correcting the Northridge earthquake record at Los Angeles – University Hospital Grounds. The test record was processed using the standard CSMIP procedure of filtering (same as procedure of Trifunac [1]). This test acceleration record does not have any long periods, and when integrated twice does not produce residual displacement (Figure 2c).

The ideal test acceleration record (Figure 2a, blue line) was contaminated by tilt record shown in Figure 2b. The maximum amplitude of tilt was equivalent to  $0.6^{\circ}$ , and had a simple shape of one period of sinusoid (with a period of  $\sim 0.7$  sec). Figure 2a shows both ideal acceleration (dark line) and tilt item ( $\sigma\psi$ ). The maximum amplitude of tilt item is about 2% of maximum acceleration.

The displacement calculated by double integration of the acceleration record contaminated by tilt (ideal record + tilt record) produces a curve (Figure 2c) that looks like a perfect case of displacement with residual displacement.

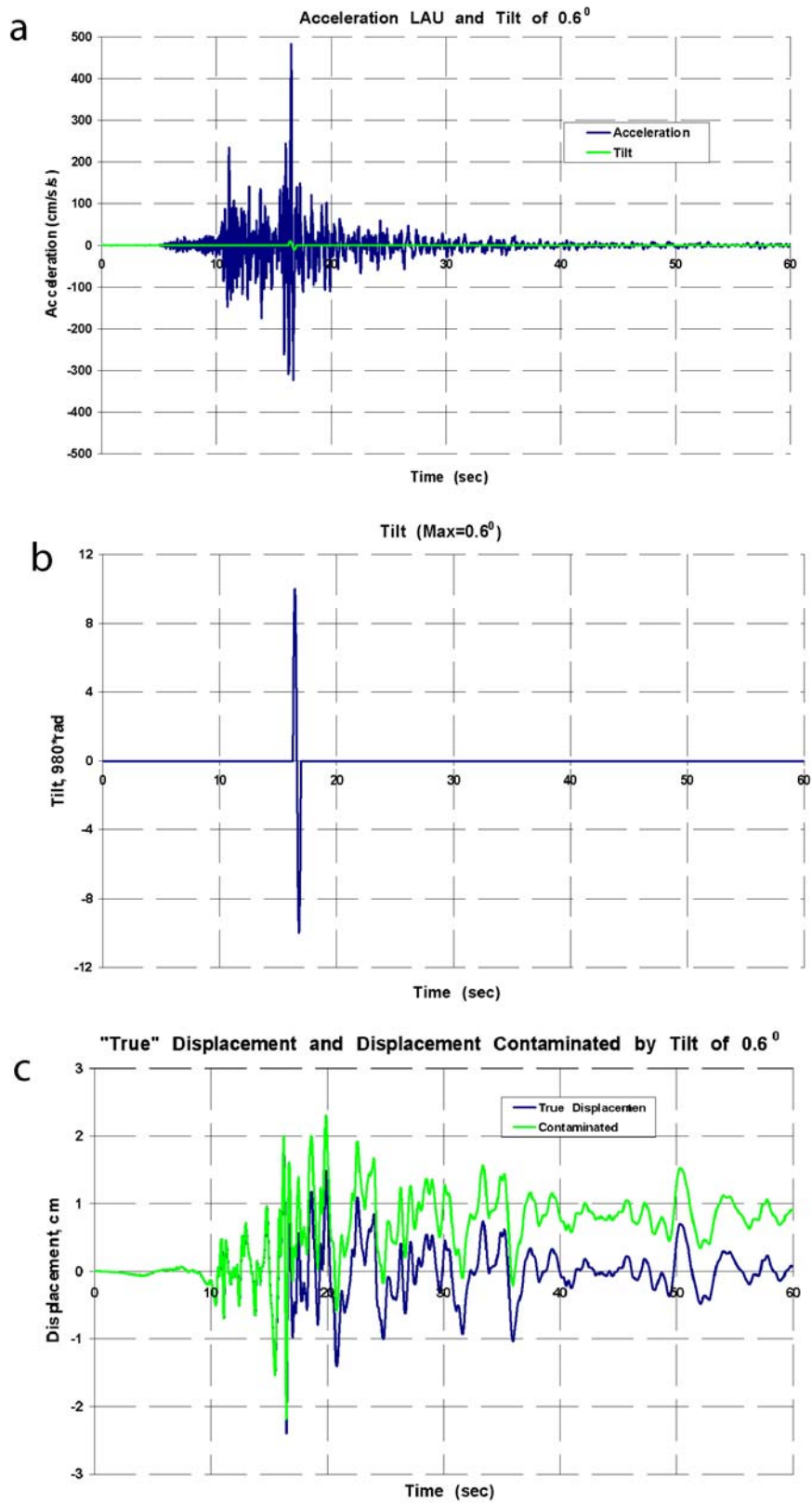
Results of this experiment show that tilting of the instrument during the earthquake motion can result in the same effect as residual displacement of the ground. If tilting occurs during the highest motion it may look like realistic residual ground displacement.

The second test is shown in Figure 3, and was constructed in the following way: The record that possibly contains residual displacement (Figure 3a) was also contaminated by tilt. This record was created by using the corrected displacement curve from the Hector station during the Hector Mine earthquake. It was assumed that this specific solution is an ideal ground motion with the corresponding ideal acceleration shown in Figure 3a (dark line).

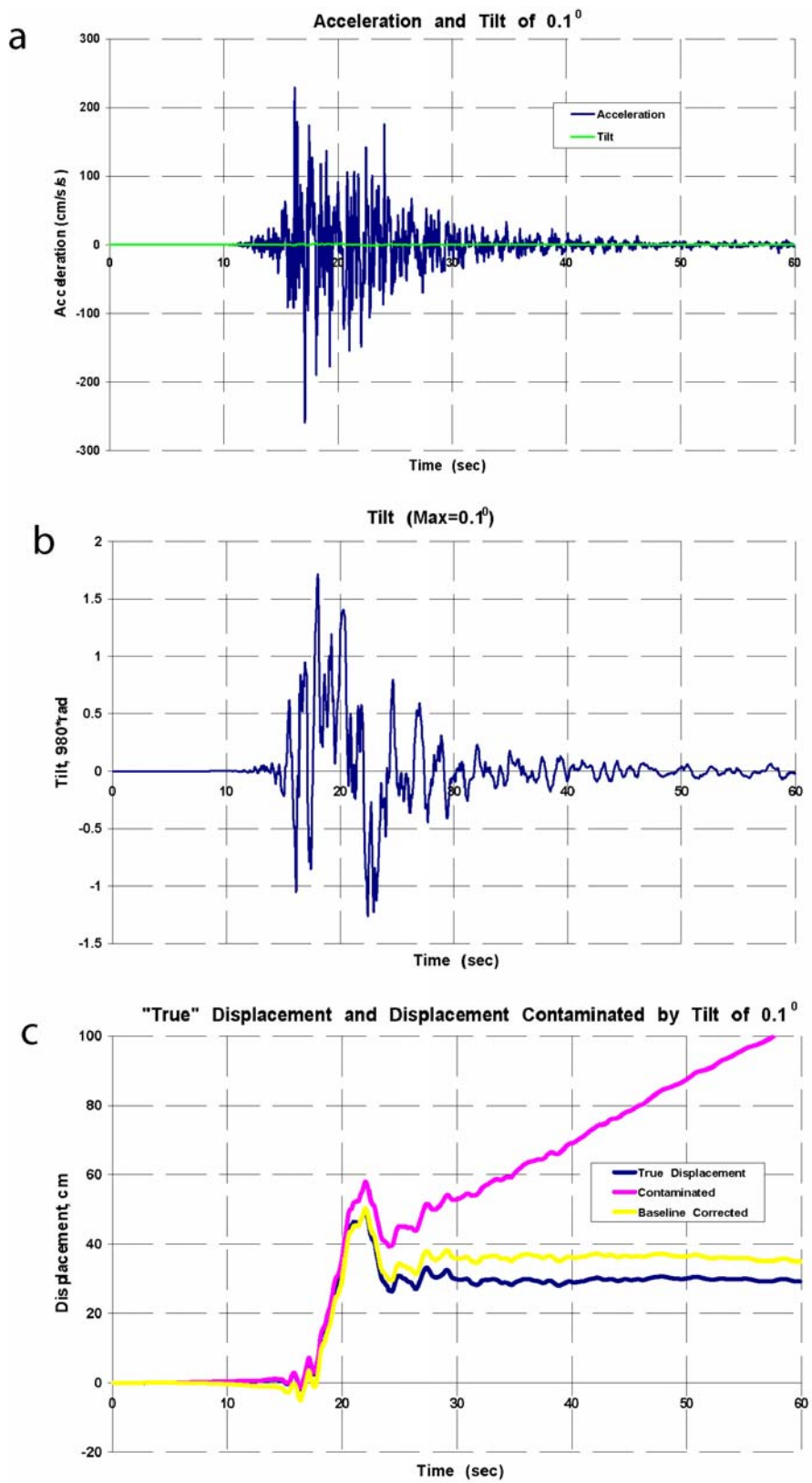
The tilt record was generated based on the assumption that the tilt spectrum is proportional to the ground velocity spectrum (Trifunac, Todorovska [23]). The ground velocity curve was normalized to the maximum amplitude corresponding to the tilt of  $0.1^{\circ}$ . In this case maximum amplitude of tilt motion ( $\sigma\psi$ ) was less than 1% than maximum acceleration.

Figure 3c shows the three following curves:

1. True or ideal displacement (dark blue line),
2. Displacement obtained by double integration of true acceleration contaminated by tilt,
3. Displacement calculated using Graizer's algorithm of baseline correction [5].



**Figure 2. Comparison of the “true” displacement and displacement calculated using accelerogram contaminated by tilt.**



**Figure 3. Comparison of the “true” displacement and displacement calculated using accelerogram contaminated by tilt.**

Application of the baseline correction algorithm makes the displacement solution look very real. It produces an error of about 21% in residual displacement calculation.

These tests results are applicable to the records of the horizontal sensors. The tests shown in Figures 2 and 3 clearly demonstrate that tilts can contaminate results of ground motion calculations with residual displacement. Their influence may result in non-reliable residual displacement.

Tilt influence can also result in differences in the long-period component of the horizontal and vertical records, since the vertical pendulum is much less sensitive to tilts than the horizontal ones.

Only records of vertical sensors (for tilts less than  $10^0$ ) can be used for residual displacement calculations.

Basically, only a six-component system or a three-component system in combination with gyroscopes (similar to those used in inertial navigation) allow reliable measurements of residuals using accelerograms.

Coming back to the analysis of existing three component accelerograms, it is possible to conclude that conservative procedure developed by Trifunac and Lee [1,2] and other similar ones are the only way for routine processing of existing strong-motion data.

## **CONCLUSIONS**

Commonly used in seismological and earthquake engineering practice, strong-motion instruments are sensitive not only to the translational motion, but also to tilts. This sensitivity can be neglected in far-field measurements, but not in the near-field studies. Numerical experiments demonstrate that ignoring tilt effects in strong-motion studies can introduce long-period error, especially for calculation of residual displacements.

In contrast to horizontal sensors, vertical sensors are practically not sensitive to tilts. This makes them potentially more usable for the long-period and residual displacement calculations.

Conservative methods of strong-motion data processing that involve filtering in a limited frequency band have a clear advantage, especially for routine data processing, because they are getting rid of the long-period component partially introduced by tilting.

It seems to be desirable to start measuring the rotational component of the strong-ground motion in combination with classical translational motion measurements in the vicinity of the faults.

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